by a *Mad Dog*, or hot Fomentations, might be of greater Service than cold Applications: For a cold Bath ſhuts the Pores, as a warm one opens them; therefore the Blood being allowed to be greatly inflamed in this Caſe, and *Dampier*'s *Powder* being a very hot Medicine, it is reaſonable to think, that when a Patient takes it, the ſetting him up to the Chin in hot Water for ſome Hours, would help the Operation of the *Powders*, by diluting the Blood, and relaxing the Pores.

VII. *A brief Account by Mr.* John Eames, *F. R. S. of a Work entitled,* The Method of Fluxions and Infinite Series, with its Application to the Geometry of Curve Lines, *by the Inventor Sir* Iſaac Newton, *Kt.* &c. *Tranſlated from the Author's Latin Original not yet made publick. To which is ſubjoin'd a perpetual Comment upon the whole,* &c. *by* John Colſon, *M. A.* and *F. R. S.*

THIS Poſthumous Work of our late excellent Preſident, a Tranſlation of which we have now received from the Hand of the learned and ingenious Mr. *Colſon*, has been long and impatiently expected by the curious in theſe Matters; and now it appears, I believe it will fully anſwer, if not exceed, thoſe Expectations, as well as confirm the Reputation the Author

thor has so justly acquired by his other Writings. For it is wrote with the same Genius and *Acumen*, it explains the Principles of his Method of Fluxions with great Clearness and Accuracy, and applies those Principles to very general and scientifical Speculations in the higher Geometry. And farther to explain this Work, and to supply such Things, for the Use of common Readers, which the Author, according to his usual Brevity, has often omitted; the Translator has thought fit to give us a Comment on a good Part of the Work, and has promised the rest at a proper Season. His Fitness for such an Undertaking is well known to the learned World, into which he was many Years ago introduced by a very good Judge, as a Person who was *reconditioris Analyseos peritissimus*.

This Text may very well be divided into three Parts: An Introduction, containing the Method of Infinite Series; The Method of Fluxions and Fluents; and lastly, The Application of both to the most considerable Problems of the higher Geometry. The Comment consists of very valuable and curious Annotations, Illustrations, and Supplements, in order to make the whole a *compleat Institution* for the Use of Learners. I shall take a kind of comparative View of the Text and Comment together.

The great Author, in what is called the Introduction, teaches the Rudiments of his Method of Infinite Converging Series, which is preparatory to that of Fluxions. In this he shews how all Compound Algebraical Quantities may be resolved into Series of simple Terms, which will converge to those compound Quantities, or rather to their Roots; just as in com-

mon Decimal Arithmetick, any complicate Number whatever, rational or ſurd, may be proſecuted and exhibited to what Degree of Accuracy we pleaſe, by decimal Parts continued *in infinitum*. And this general Arithmetick is here applied to the finding of the Roots of all kinds of Algebraical Equations, whether pure or affected.

And this Doctrine is carried on ſtill farther by Mr. *Colſon* in his Comment. He purſues the Author's Hint, that vulgar Arithmetick and Algebra, decimal Fractions and infinite Series, have the ſame common Foundation, and compoſe together but one uniform Science of Computation. For, as in our vulgar Arithmetick, when rightly explain'd, we expreſs and compute all Numbers by the Root *Ten*, and its ſeveral Powers and their Reciprocals, together with a Set of certain known and ſmall Coefficients; ſo in this more univerſal Arithmetick of Infinite Series, we do the ſame thing in effect, by means of any Root aſſumed at Pleaſure, its Powers and their Reciprocals, diſpoſed in a regular deſcending Order, together with any Coefficients, as it may happen. And when theſe Series duly converge, they will as truly exhibit by their Aggregate the Quantity required, as a Decimal Fraction infinitely continued will approximate to its proper *Quæſitum*. This gives him Occaſion to expatiate largely upon the Nature and Conſtruction of Arithmetical Scales, particular and general; and to inquire into the Nature and Formation of Infinite Series, and their Circumſtances of Convergency and Divergency. To explain which he ſhews, that in every Series there is always a Supplement to be underſtood, when it is not exhibited. This Supplement ſums up the Series, and makes

makes it ſtop at a finite Number of Terms, in Series that either converge or diverge. Whence in diverging Series it muſt neceſſarily be found and admitted, or otherwiſe the Concluſion will not be true; but in converging Series, where it can ſeldom be known, it may ſafely be omitted, becauſe it continually diminiſhes with the Terms of the Series, and finally becomes leſs than any aſſignable Quantity.

The Nature of infinite Series being thus diſplay'd, he applies them to the Reſolution of all kinds of Algebraical Equations. He explains in a very general Manner, the Author's famous Artifice, for finding the *Forms of the Series* for the Roots, and their initial Approximations, by means of a Parallelogram and Ruler, and ſhews its Application in all Caſes. Then he invents many ways of Analyſis, by which the Roots are further proſecuted, and may be produced to any Degree of Accuracy required. Alſo many other Speculations are added, to compleat the Doctrine of Series; particularly a very general and uſeful Theorem, for the Solution of all affected Equations in Numbers.

From the Reſolution of Equations and the Doctrine of Infinite Series, which finiſhes the firſt Part of this Work, Sir *Iſaac Newton* proceeds to lay down the Principles of his Method of Fluxions, which is the chief Deſign of the preſent Treatiſe. This Method he founds upon the abſtract or rational Mechanicks, by ſuppoſing all Mathematical Quantities to be generated, as it were, by local Motion, and therefore to have relative Velocities of Increaſe or Decreaſe, which Velocities he calls *Fluxions*. And the Quantities ſo generated by a continual Flux he calls *Flu-*

ents or flowing Quantities; the Relation of which Fluents is always exprefs'd by fome Algebraical Equation, either given or required. If this Equation be given, and the Relation of the Fluxions is required, it conftitutes the *direct Method of Fluxions*; but when the contrary, 'tis the *inverfe Method of Fluxions.*

Sir *Ifaac*, in his firft Problem, which takes in the direct Method of Fluxions, fhews how to find the Relation of the Fluxions in a very general Manner, and by a great Variety of Solutions. This way of refolving the Problem is peculiar to this Work. He likewife extends it to Equations involving feveral Fluents, which accommodates it to thofe Cafes, wherein any complex or irrational Quantities may be found, or Quantities that are geometrically irreductible. Then he demonftrates the Principles of his Method, or the Precepts of Solution, from the Nature of Moments or vanifhing Quantities, and from the obvious Properties of Equations, which involve indeterminate Quantities.

The Commentator much inlarges upon this whole Doctrine; he enters into the Reafon and Ufe of this Multiplicity of Solutions, and fhews it is a neceffary Refult from the different Forms the fame given Equation may acquire. But efpecially he takes the Author's Demonftration into ftrict Examination, endeavours farther to illuftrate and enforce its Evidence, and to clear it from all the Objections that either have or may be urged againft it. He even contends, that tho' the Moments and vanifhing Quantities of the Author could be proved to be impoffible, as has been fuggefted by fome Mathematicians, yet even then they would

would be ſufficient for all the purpoſes of Fluxions, and he produces Inſtances of a like Nature from other parts of Mathematicks. And tho' the Author, Sir *Iſaac Newton*, in his preſent Treatiſe, does not directly mention ſecond Fluxions, or thoſe of higher Orders; yet the ingenious Commentator thinks proper to extend his Inquiries to theſe Orders of Fluxions, demonſtrates their Theory, gives Rules and Examples for deriving their Equations, proves their relative Nature, and even exhibits them to View by Geometrical Figures. This laſt he does chiefly in what he calls the Geometrical and Mechanical Elements of Fluxions; and he contrives a very general Method, by means of Curve-lines and their Tangents, to make Fluxions and Fluents the Objects of Senſe and ocular Inſpection; and thereby he illuſtrates and verifies the received Methods of deriving their Equations in all Caſes.

In the Author's ſecond Problem, or the Relation of the Fluxions being given to determine the Relation of the Fluents, which includes the inverſe Method of Fluxions, he begins with a particular Solution of it. He calls this Solution particular, becauſe it extends only to ſuch Caſes, wherein the given Fluxional Equation either has been, or might have been, derived from ſome previous finite Algebraical Equation. Then he ſhews how we may return directly to this Equation. But this is ſeldom the Caſe of ſuch Fluxional Equations, whoſe Fluents or Roots are propoſed to be found. For they have commonly Terms either redundant or deficient, by which they cannot be brought under this particular Solution. Therefore to anſwer this Caſe alſo, he gives us a general Solution, in which he extracts the Roots of any

propoſed

proposed Fluxional Equation, by several ingenious Methods of Analysis. And here it is chiefly, that he calls his Method of Infinite Series to his Assistance; for the Fluent, or Root, will here always be exhibited by a Series. And to find the Fluent in finite Terms, when it can be done, requires particular Expedients, as we shall see afterwards.

Mr. *Colson* in his Comment upon this Part of the Work is very full and explicit. He explains and applies the Author's particular Solution; but is much more copious in explaining the Examples, and clearing up the Difficulties and Anomalies of the general Solution. This is chiefly perform'd by introducing several new and simple Methods of Analysis, or Processes of Resolution; and by applying the Author's Artifice of the Ruler and Parallelogram mention'd before, to these Fluxional Equations: By which means not only the Forms of the Series are determin'd, and their initial Approximations, as has been observ'd above; but likewise all the Series may be found, that can be derived from the same Fluxional Equation. The Commentator concludes by giving us a very general Method for resolving all Equations, whether Algebraical or Fluxional; which Method requires no foreign Assistance, or no subsidiary Operations, which all other Methods do. It is founded upon the Use and Admission of the higher Orders of Fluxions, and is exemplify'd by the Solution of several useful Problems. Here the Comment leaves us, but we will go on with our Author.

Having thus taught us the Method of Fluxions, both direct and inverse, he proceeds to apply this Method to some very curious and general Problems,

chiefly in the Geometry of Curve-lines. As firſt, he determines the *maxima* and *minima* of Quantities in all Caſes, and propoſes ſome elegant Problems to illuſtrate this Doctrine. Then he teaches us to draw Tangents to Curves, whether Geometrical or Mechanical, and that after a great Variety of Ways, or however the Nature of the Curve may be defined. Here likewiſe he propoſes ſome Queſtions, to exerciſe and improve the Learner: Then is very particular upon finding the Quantity of Curvature, at any Point of a given Curve, whether Geometrical or Mechanical, or in determining the Centre and the Radius of Curvature: To which ſeveral other curious Speculations are ſubjoin'd of a like Nature. Here he communicates a very elegant and intirely new Problem, for determining the Quality of the Curvature, at any Point of a given Curve; or how the Curvature proceeds in reſpect of its greater or leſs Inequability.

Afterwards he goes on to the Quadrature of Curves, which chiefly gives occaſion to apply the inverſe Method of Fluxions. And firſt he ſhews how, by the direct Method, to find as many Curves as you pleaſe, (or to determine their Equations) the Areas of which ſhall be capable of an exact Quadrature. Then he ſhews how to find as many Curves as you pleaſe, which, tho' not capable of a juſt Quadrature, yet their Areas may be compared to thoſe of the Conic Sections, or of ſuch other Curves as ſhall be aſſign'd. Laſtly, He ſhews how to determine in general the Area of any Curve that ſhall be propoſed, chiefly by the Method of Infinite Series; where many curious and uſeful Speculations are occaſionally introduced and inſerted: As how to aſcertain the Limits of an

Area,

Area, when thus found analytically; how commodiously to square the Circle, the Ellipsis, or Hyperbola, and how to apply the Quadrature of this last to the computing a Canon of Logarithms; the Construction of Tables for the ready finding of Quadratures, or the Comparison of Areas, and how to apply them to the solving of other like Problems; the forming of Constructions, and demonstrating Theorems by Fluxions; the approximating to Areas mechanically, and such-like.

From finding of Areas he proceeds to the *Rectification of Curves*; and first he shews how to find as many Curves as you please, whose Curve-lines are capable of an exact Rectification. Then he teaches us to find as many Curves as we please, whose Curve-lines, tho' not capable of a just Rectification, yet may be compared with the Lengths of any Curve-lines assign'd, or with the Areas of any Curve, when reduced to the Order of Lines. Lastly, He determines the Lengths of any Curve in general, and gives several proper Examples of it. All which elegant Speculations are managed with admirable Skill, great Subtilty, and fine Contrivance.